

The dilaton as a candidate for dark matter

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Abstract: We examine consequences of the stabilization of the dilaton through the axion. An estimate of the resulting dilaton potential yields a relation between the axion parameter $m_a f_{PQ}$ and the average instanton radius, and predicts the ratio between the dilaton mass m_ϕ and the axion mass m_a . If we identify the string axion with a Peccei–Quinn axion, then $m_\phi m_{Pl} \sim m_a f_{PQ}$, and the dilaton should be strongly aligned $\sqrt{\langle \phi^2 \rangle} \leq 10^{-4} m_{Pl}$ at the QCD scale, in order not to overclose the universe.

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There exists strong experimental and theoretical evidence for the existence of dark matter in the universe from galactic rotation curves, gravitational lensing, and standard inflationary models. An exciting possibility for theoreticians and experimentalists alike is the existence of non-baryonic dark matter. Investigation of properties of non-baryonic dark matter and its cosmological implications is a very interesting topic on the forefront both of astrophysics and particle physics, and concentrates mainly on massive neutrinos, neutralinos as harbingers of supersymmetry, and axions, which are predicted from the Peccei–Quinn mechanism to solve the strong CP problem in QCD [1, 2, 3] and appear also in string theory [4]. The books of Kolb and Turner and of Börner provide very useful reviews and critical discussions of dark matter in cosmology [5, 6].

The axion is one of the leading candidates for cold dark matter in the universe (besides the lightest supersymmetric particle) and very likely made an important contribution to structure formation in the universe¹. However, string theory and any theory involving a Kaluza–Klein picture of physics beyond the standard model also predict a dilaton, with a characteristic coupling to gauge fields of the form $\exp(\lambda\phi)F^2$. Here ϕ denotes the dilaton and λ is a characteristic length, typically of the order of the Planck length², which governs all non-gravitational interactions of the dilaton. Indeed, recent results on strong–weak coupling dualities in gauge theories and string theory provide strong evidence that the axion and the dilaton should come together, and that their properties are intimately connected. From these features, it is very likely that the dilaton may also serve as a suitable candidate for cold dark matter, not as a competitor but as a companion of the axion. Unfortunately, discussions of cosmological implications of a dilaton suffer from the problem of dilaton stability.

String inspired effective field theories and Kaluza–Klein theories have an invariance of the equations of motion under constant shifts of the dilaton $\phi \rightarrow \phi + c$ and appropriate rescalings of the other fields. Even worse, if the field theory is supersymmetric the non-renormalization theorem tells us that no dilaton mass can be generated perturbatively [8]. This is worrisome, since shifts of the dilaton correspond to rescalings of couplings and masses in the field theory and question the meaning of these parameters. This problem is known as the dilaton problem.

Investigations in the subject of dilaton stabilization concentrate on the construction of appropriate super-potentials or Kähler-potentials which stabilize $\langle \exp(\lambda\Phi) \rangle$ at phenomenologically interesting expectation values, often linking the condensate to a gaugino condensate and supersymmetry breaking. [9] contains a list of pioneering papers on this subject. Duality invariant gaugino condensation and S-duality invariant super-potentials were investigated in [10, 11], and recent contributions can be

¹See [7] for a recent review.

²Our conventions for Planck units are $m_{Pl} = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{GeV}$.

found in [12], where the coupling of the chiral dilaton multiplet is re-examined, and in [13], where the dilaton is treated in the linear multiplet.

In the present paper I will point out that topologically non-trivial configurations of the axion and instantons generate a dilaton potential and consider some consequences of this observation [14]. However, for the time being I will not examine what kind of super-potential this mechanism might generate in four-dimensional supersymmetric field theories, but concentrate on the dilaton potential and its cosmological implications.

The main players in the game are a dilaton ϕ , an axion a and gauge fields A_μ with field strengths $F_{\mu\nu}$, and their mutual interactions before taking into account non-perturbative effects are governed by a Lagrangian

$$\begin{aligned} \frac{1}{\sqrt{-g}}\mathcal{L} = & \frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi - \frac{1}{2}\exp(-2\lambda\phi)g^{\mu\nu}\partial_\mu a \cdot \partial_\nu a \\ & - \frac{1}{4}\exp(\lambda\phi)F_{\mu\nu}^j F_j^{\mu\nu} + \frac{q^2}{64\pi^2 f_{PQ}}\epsilon^{\mu\nu\rho\sigma}a F_{\mu\nu}^j F_{\rho\sigma j} \end{aligned} \quad (1)$$

In writing down this Lagrangian, we explicitly split the dilaton into its expectation value $\langle\exp(\lambda\Phi)\rangle \equiv q^{-2}$ and the fluctuations ϕ around the expectation value. We also rescaled the gauge fields and the axion such that the covariant derivatives are $D_\mu = \partial_\mu - igA_\mu$ and the kinetic term for the axion has standard normalization.

The dilaton-axion and dilaton-gluon couplings in (1) match in such a way that the system exhibits an $\text{SL}(2,\mathbb{R})$ strong-weak coupling duality in the abelian case [15]. S-duality in string theory [11, 16, 17, 18] indicates, that the low energy regime should also be described by a field theory with dilaton couplings related in this way, and it is a remarkable property of Kaluza-Klein theory that compactifications from D to d dimensions yield exactly this S-dual dilaton coupling if and only if³ $d = 4$ [19].

In the case of an abelian gauge group S-duality fixes the dilaton scale according to

$$\frac{1}{\lambda} = \frac{8\pi^2}{q^2}f_{PQ}$$

The resulting $\text{SL}(2,\mathbb{R})$ invariance of the equations of motion is most conveniently described in terms of the axidilaton

$$z = \frac{1}{f_{PQ}}[a + \frac{i}{\lambda}\exp(\lambda\phi)]$$

The symmetry is realized via

$$z' = \frac{a_{11}z + a_{12}}{a_{21}z + a_{22}}, \quad a_{11}a_{22} - a_{12}a_{21} = 1$$

³It is tempting to conclude from this property, that preservation of S-duality in the low energy regime picks out four dimensions.

$$F'_{\mu\nu} - i\tilde{F}'_{\mu\nu} = (a_{21}z + a_{22})(F_{\mu\nu} - i\tilde{F}_{\mu\nu})$$

which means that the self-dual part of the Yang–Mills curvature transforms like a half-differential on the axidilaton upper half-plane.

This duality symmetry does not survive in the nonabelian case, since the duality rotation replaces the Bianchi identity $D^\mu \tilde{F}_{\mu\nu} = 0$ by $D^\mu \tilde{F}'_{\mu\nu} = 0$, and it is not possible to identify the rotated field strengths with duality rotated gauge potentials.

Perturbatively the nonabelian theory still has a Peccei–Quinn symmetry $z \rightarrow z + a_{12}$ with a conserved current

$$\frac{1}{\sqrt{-g}}j^\mu = \exp(-2\lambda\phi)g^{\mu\nu}\partial_\nu a + \frac{q^2}{16\pi^2 f_{PQ}}\epsilon^{\mu\nu\rho\sigma}(A_\nu^j\partial_\rho A_{\sigma j} + \frac{q}{3}f_{ijk}A_\nu^i A_\rho^j A_\sigma^k)$$

and the scaling symmetry of the equations of motion, which serves as a harbinger of the dilaton problem, is also preserved if the metric is rescaled:

$$\begin{aligned}\phi &\rightarrow \phi + c \\ g^{\mu\nu} &\rightarrow \exp(-\lambda c)g^{\mu\nu} \\ A_\mu &\rightarrow A_\mu \\ a &\rightarrow \exp(\lambda c)a\end{aligned}\tag{2}$$

However, we have not yet taken into account non-trivial field configurations of the gauge fields and the axion: We infer the non-perturbative effects of these field configurations from the Lagrangian of the Euclidean action. In a flat background this takes the form:

$$\begin{aligned}\mathcal{L}_E &= \frac{1}{2}g^{\mu\nu}\partial_\mu\phi \cdot \partial_\nu\phi + \frac{1}{2}\exp(-2\lambda\phi)g^{\mu\nu}\partial_\mu a \cdot \partial_\nu a \\ &+ \frac{1}{4}\exp(\lambda\phi)F_{\mu\nu}^j F_j^{\mu\nu} + i\frac{q^2}{32\pi^2 f_{PQ}}a\tilde{F}_{\mu\nu}^j F_j^{\mu\nu}\end{aligned}\tag{3}$$

Positivity of the real part and the estimate of the effective axion potential by Vafa and Witten [20] indicate that the dominating contributions to the path integral come from instanton configurations $F = \pm\tilde{F}$ with constant dilaton and the axion frozen to integer multiples of $2\pi f_{PQ}$. This survival of instantons in the presence of the dilaton is crucial, since integrality of the instanton number and invariance of the path integral discretize Peccei–Quinn symmetry

$$\frac{a_{12}}{2\pi} \in \mathbb{Z},$$

thereby also breaking the scale invariance (2). The picture emerging from this observation shows us that instantons create an effective axion potential with an enumerable

set of equidistant vacua, thus discretizing Peccei–Quinn symmetry. Discreteness of the axion vacua and the cosine–like shape of the axion potential displayed below then implies breaking of the scaling symmetry and lifts the degeneracy of the dilaton.

The impact of instantons on the effective axion potential has been examined by several authors, and the interpretation of instantons as real time tunneling configurations between gauge theory vacua suggests

$$V(a) = m_a^2 f_{PQ}^2 \left(1 - \cos\left(\frac{a}{f_{PQ}}\right) \right) \quad (4)$$

if the instanton gas is dilute enough to neglect higher order cosine terms [21, 22, 23]. While initially this result was inferred from semiclassical calculations of tunneling amplitudes, the same potential can also be derived in a direct instanton calculation if the wavelength of the axion is large compared to the instanton size.

The effective axion potential in turn breaks the scale invariance (2) and indicates that the axidilaton–gluon system also lifts the degeneracy of the dilaton. This is obvious in the gauge sector: Instantons push the dilaton into the strong gauge coupling regime, since the action of the instantons decreases with decreasing $\langle\phi\rangle$. However, non–trivial configurations also arise in the axion sector:

- If a is periodic $a \sim a + 2\pi f_{PQ}$, then it contributes local minima to the Euclidean path integral over $\exp(-S_E)$ in the form of *axion walls* (instead of axion strings in three dimensions). Periodicity of a arises, if it is related to the argument of a complex field with frozen modulus in the low energy regime. This is e.g. the case, if a arises as the phase of a determinant of local fermion masses.
- If a is not an angular variable, then all the possible vacua $\langle a \rangle = 2\pi f_{PQ} n$ are distinct and we expect three–dimensional domain walls separating four–dimensional domains where a approximates different vacua.

Both of these topological defects mark regions of non–vanishing gradients ∂a and favor large values of the dilaton through the dilaton–axion coupling, thus compensating the effect of the instantons.

From these observations we may infer an estimate of the dilaton potential, if we define a characteristic length Δ of the axion defects: In the case of an angular axion Δ would measure the circumference of the axion walls, while in the second case the four–dimensional domain boundaries are extended in three dimensions and have an average thickness Δ in the fourth direction. If ϱ denotes both the average extension and separation of instantons we find an estimate for the effective dilaton potential

$$V(\phi) = \frac{48}{q^2 \varrho^4} \exp(\lambda\phi) + 2\pi^2 \frac{f_{PQ}^2}{\Delta^2} \exp(-2\lambda\phi) \quad (5)$$

and this implies for the gauge coupling

$$\langle \exp(\lambda\Phi) \rangle = \frac{1}{q^2} = \frac{\pi^2 f_{PQ}^2 \varrho^4}{12\Delta^2} \quad (6)$$

The potential (5) then yields a dilaton mass

$$m_\phi = \frac{12\lambda}{q\varrho^2} \quad (7)$$

If a is not an angular variable we may estimate the parameter Δ by minimizing the energy density of the axion domain spaces

$$u = 2\pi^2 \frac{f_{PQ}^2}{\Delta} + m_a^2 f_{PQ}^2 \Delta. \quad (8)$$

This yields a thickness of the order

$$\Delta \simeq \frac{\pi\sqrt{2}}{m_a} \quad (9)$$

which is of the same order as the thickness of ordinary axion domain walls in Minkowski space [5]. From (6) and (9) we find a relation between the axion parameters and the average instanton radius

$$m_a^2 f_{PQ}^2 \simeq \frac{6}{\pi\alpha_q \varrho^4} \quad (10)$$

The average instanton radius is set by the QCD scale [22]

$$\frac{1}{\varrho} \sim \Lambda_{\text{QCD}} \sim 2 \times 10^8 \text{eV}$$

whereas from [24, 25] we learn that

$$m_a f_{PQ} \sim m_\pi f_\pi \sim 10^{16} \text{eV}^2$$

Relation (10) is in gross agreement with these estimates if $\alpha_q \sim O(10)$.

The potential (5) then implies a relation between the dilaton mass and the axion mass

$$m_\phi \simeq \sqrt{6}\lambda m_a f_{PQ} \quad (11)$$

which hints at a non-perturbatively generated dilaton mass which is much smaller than the axion mass:

$$m_\phi \sim 10^{-6} m_a$$

However, assuming $\lambda \simeq m_{Pl}^{-1}$, we encounter a fine-tuning problem: The dilaton behaves very similar to a misalignment produced light axion, and the estimates of the cosmic abundance of the axion [26, 27, 28, 29] carry over to the dilaton with some minor modifications. Since the temperature where a misaligned field starts to oscillate goes with $T \sim m^{0.18}$ [29, 5], the dilaton will start to oscillate after the axion, when the temperature has dropped by another factor of 10. Then we find for the dilaton contribution to the critical density of the universe

$$\frac{\Omega_\phi}{\Omega_a} \simeq 10^{-5} \frac{\langle \phi^2 \rangle}{f_{PQ}^2} \simeq 10^7 \frac{\langle \phi^2 \rangle}{m_{Pl}^2} \quad (12)$$

In the derivation of this ratio we assumed that the dilaton is non-relativistic at the onset of oscillations. This is a justified assumption, since the drop in the temperature relative to the onset of the axion oscillations implies for the ratio of the momenta $p_\phi \simeq 10^{-2} p_a$, yielding a velocity $p_\phi/m_\phi \simeq 10^{-2}$.

Imposing a bound $\Omega_\phi/\Omega_a \leq 0.1$ then implies fairly strong alignment of the dilaton:

$$\sqrt{\langle \phi^2 \rangle} \leq 10^{-4} m_{Pl}$$

Eventually, this could be attributed to inflation of a local patch with a small variance of the dilaton, which would also explain the absence of variations in spectral lines over cosmic distances: Patches with small variation of the dilaton survive and inflate, while overclosed patches collapse. The variation of the dilaton in the inflating patch remains small, because the dilaton is very weakly coupled.

We have encountered a generic problem of Planck scale physics: Light moduli either must have a decay constant far below the Planck scale, or they must be strongly aligned when oscillation begins to dominate, in order not to overclose the universe. Recent proposals for accomodating very small decay constants in terms of Planck units are discussed in [30] (see also [31]), and eventually we may also find a mechanism to considerably lower the decay constant of the dilaton. This would not alter our conclusions about the mechanism of dilaton stabilization, but could imply a more conventional picture of the cosmological significance of the dilaton.

References

- [1] R.D. Peccei and H.R. Quinn, *Phys. Rev. Lett.* **38** (1977) 1440; *Phys. Rev.* **D16** (1977) 1791.
- [2] S. Weinberg, *Phys. Rev. Lett.* **40** (1978) 223.
- [3] F. Wilczek, *Phys. Rev. Lett.* **40** (1978) 279.

- [4] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, 2 Vols., Cambridge University Press, Cambridge 1987.
- [5] E.W. Kolb and M.S. Turner, *The Early Universe*, Addison–Wesley, Reading 1990.
- [6] G. Börner, *The Early Universe*, 3rd ed., Springer, Berlin 1993.
- [7] G.G. Raffelt, *Axions in astrophysics and cosmology*, hep-ph/9502358.
- [8] M. Dine and N. Seiberg, *Phys. Rev. Lett.* **57** (1986) 2625.
- [9] G. Veneziano and S. Yankielowicz, *Phys. Lett.* **B113** (1982) 231;
S. Ferrara, L. Girardello and H.P. Nilles, *Phys. Lett.* **B125** (1983) 457;
J.P. Derendinger, L.E. Ibañez and H.P. Nilles, *Phys. Lett.* **B155** (1985) 467;
M. Dine, R. Rohm, N. Seiberg and E. Witten, *Phys. Lett.* **B156** (1985) 55;
T.R. Taylor, *Phys. Lett.* **B164** (1985) 43.
- [10] A. Font, L. Ibañez, D. Lüst and F. Quevedo, *Phys. Lett.* **B 245** (1990) 401.
- [11] A. Font, L. Ibañez, D. Lüst and F. Quevedo, *Phys. Lett.* **B249** (1990) 35.
- [12] Z. Lalak, A. Niemeyer and H.P. Nilles, *Nucl. Phys.* **B453** (1995) 100.
- [13] P. Binétruy, M.K. Gaillard and Y.-Y. Wu, *Dilaton stabilization in the context of dynamical supersymmetry breaking through gaugino condensation*, hep-th/9605170.
- [14] R. Dick, *Stabilizing the dilaton through the axion*, LMU–TPW 96–22, submitted to *Mod. Phys. Lett.* **A**.
- [15] A. Shapere, S. Trivedi and F. Wilczek, *Mod. Phys. Lett.* **A6** (1991) 2677.
- [16] J.H. Schwarz and A. Sen, *Nucl. Phys.* **B411** (1994) 35; *Phys. Lett.* **B312** (1993) 105.
- [17] A. Sen, *Int. J. Mod. Phys.* **A9** (1994) 3707; *Phys. Lett.* **B329** (1994) 217.
- [18] J.H. Schwarz, *Lectures on superstring and M theory dualities*, hep-th/9607201.
- [19] R. Dick, *The dilaton in gravity and cosmology*, in preparation.
- [20] C. Vafa and E. Witten, *Phys. Rev. Lett.* **53** (1984) 535.
- [21] S. Coleman, in *The Whys of Subnuclear Physics*, A. Zichichi (Ed.), Plenum Press, New York 1979, pp. 805–916.

- [22] D.J. Gross, R.D. Pisarski and L.G. Yaffe, *Rev. Mod. Phys.* **53** (1981) 43.
- [23] J.E. Kim, *Phys. Rep.* **150** (1987) 1.
- [24] W.A. Bardeen and S.-H.H. Tye, *Phys. Lett.* **B74** (1978) 229.
- [25] J. Kandaswamy, P. Salomonson and J. Schechter, *Phys. Rev.* **D17** (1978) 3051;
Phys. Lett. **B74** (1978) 377.
- [26] J. Preskill, M.B. Wise and F. Wilczek, *Phys. Lett.* **B120** (1983) 127.
- [27] L.F. Abbott and P. Sikivie, *Phys. Lett.* **B120** (1983) 133.
- [28] M. Dine and W. Fischler, *Phys. Lett.* **B120** (1983) 137.
- [29] M.S. Turner, *Phys. Rev.* **D33** (1986) 889.
- [30] T. Banks and M. Dine, *The cosmology of string theoretic axions*, hep-th/9608197.
- [31] K. Choi and J.E. Kim, *Phys. Lett.* **B176** (1986) 103.